MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2018/2019

DEM5018 – ENGINEERING MATHEMATICS 1

(Diploma in Electronic Engineering)

29 MAY 2019
9:00 a.m. - 11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **3** pages (excluding cover page and Appendix).
2. Answer **ALL FOUR** questions.
3. Write your answers in the answer booklet provided.
4. Formulae are provided at the Appendix section.

QUESTION 1

(a) Solve the following inequalities

i) $x^2 + 2x - 15 \leq 0$ [4 marks]

ii) $|5x + 4| \leq 6$ [3 marks]

(b) Find the equation of a straight line passing through the point $(2,4)$ and parallel to the line $2y - 3x = 8$. Then sketch the graph. [6 marks]

(c) A pebble is dropped into a calm pond at coordinates (h, k), creating a circular ripple effect with a radius of the outer ripple increases at a rate of 2 cm/s. Write the equation of the circle (of the outer ripple) after 10 seconds. [4 marks]

(d) Solve the following system by using inverse matrix method. [8 marks]

$$2x + 3y + z = 2$$
$$3x + 3y + z = -2$$
$$2x + 4y + z = 2$$

QUESTION 2

(a) Determine the exact value for $\cos^2 x + \cos x = \sin^2 x,\ 0° \leq x < 360°$. [7 marks]

(b) Verify the identity $\frac{\cos x}{\sin^2 x} - \cos x \cot^2 x = \cos x$. [5 marks]

(c) Given the complex number $s = 1 - 2i$.

(i) Convert the complex number into polar form. [4 marks]

(ii) Evaluate for s^6 and leave the answer in rectangular form. [5 marks]

(iii) Given that $t = 2 - 2i$. Find the expression for $\frac{s}{t}$ in rectangular form. [4 marks]

Continued ...

QUESTION 3

(a) A function f is defined as $f(x) = \frac{x^2+4x-12}{x^2-2x}$.

(i) Find the simplified form of $f(x)$. Hence sketch a graph for $f(x)$ where $x > 0$. [4 marks]

(ii) Determine $f^{-1}(x)$. [4 marks]

(iii) Determine $\lim_{x\to 2} f(x)$. [2 marks]

(iv) Given that $g(x) = x^2 - 2$. Find for

1. $(g \circ f)(x)$ [3 marks]

2. $(gf)(x)$ [2 marks]

(b) An airplane flying horizontally at an altitude of 3 km at a speed of 480 km/hour passes directly above an observer on the ground at time t = 0 s, as shown in Figure Q3.1. Find the rate of change of distance between the observer and airplane ($\frac{dx(t)}{dt}$ in km/hour) at the instant t = 30 s.

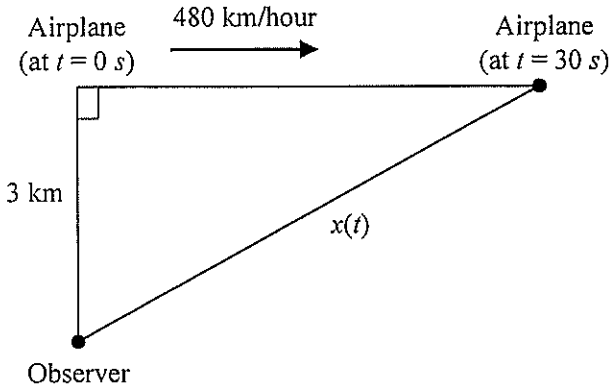

Figure Q3.1

[10 marks]

Continued...

QUESTION 4

(a) Differentiate the following functions.

(i) $y = \ln(5x^2 + 4x + 2)$ by using the chain rule. [5 marks]

(ii) $y = (4x^4 + 2x^3 - 1)(x^2 + 1)$ by using the product rule. [5 marks]

(b) Given $f(x) = x^3 + 3x^2 - 5$. Find the intervals in which the function is increasing or decreasing. [5 marks]

(c) A piece of wire 50 cm long is bent into the shape of a rectangular enclosure with a 2 cm opening, as shown in Figure Q4.1.

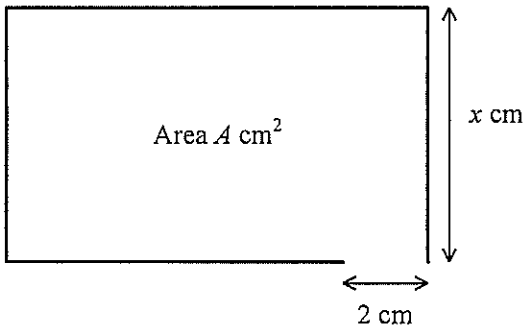

Figure Q4.1

(i) Show that the area of the enclosure is given by: $A = 26x - x^2$. [4 marks]

(ii) Find the value of x that gives the maximum area. [3 marks]

(iii) Calculate the maximum area. [3 marks]

End of paper.

APPENDIX

A Standard Form of Circle Equation

$$(x-h)^2+(y-k)^2=r^2$$

B Fundamental Identities

- Even & Odd Properties:
$$\sin(-\theta)=-\sin\theta \qquad \csc(-\theta)=-\csc\theta$$
$$\cos(-\theta)=\cos\theta \qquad \sec(-\theta)=\sec\theta$$
$$\tan(-\theta)=-\tan\theta \qquad \cot(-\theta)=-\cot\theta$$
- Reciprocal identities: $\csc\theta=\dfrac{1}{\sin\theta}$, $\sec\theta=\dfrac{1}{\cos\theta}$, $\cot\theta=\dfrac{1}{\tan\theta}$
- Pythagorean identities :
$\sin^2\theta+\cos^2\theta=1$, $\tan^2\theta+1=\sec^2\theta$, $1+\cot^2\theta=\csc^2\theta$
- Sum/Difference Formula for Cosine:
$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$
- Sum/Difference Formula for Sine:
$\sin(A\pm B)=\sin A\cos B\pm\sin B\cos A$
- Sum/Difference Formula for Tangent:
$\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$
- Double Angle Formula:
$$\cos 2A=\cos^2 A-\sin^2 A$$
$$=1-2\sin^2 A$$
$$=2\cos^2 A-1$$
$$\sin 2A=2\sin A\cos A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$
- Half Angle Formula:
$$\sin\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

C Relations between Polar and Rectangular Coordinates

$x = r\cos\theta$; $y = r\sin\theta$; $r = \sqrt{x^2 + y^2}$; $\tan\theta = \frac{y}{x}$

D Products and Quotients in Polar Form

- $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]; \quad z_2 \neq 0$

E DeMoivre's Theorem

- Powers of Complex Numbers on Polar Form:
  $z^n = r^n(\cos n\theta + i\sin n\theta)$

- Roots of Complex Numbers in Polar Form:
  $z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)\right]$
  $k = 0, 1, 2, \ldots, n-1$

F Derivatives of Trigonometric Functions

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

G The Chain Rule

If $y = f[g(x)]$ and $u = g(x)$, then $y = f(u)$:

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ $\quad$ *or* $\quad$ $\frac{d}{dx}[f(u)] = f'(u).\frac{du}{dx}$

H Derivative of the Exponential & logarithmic Function

$$\frac{d}{dx}[e^x] = e^x; \qquad \frac{d}{dx}[e^u] = e^u.\frac{du}{dx}; \qquad \frac{d}{dx}\left(a^x\right) = a^x \ln a$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0 \quad ; \qquad \frac{d}{dx}[\ln u] = \frac{1}{u}.\frac{du}{dx};$$

$$\frac{d}{dx}\log_a x = \frac{1}{\ln a}\cdot\frac{1}{x} = \frac{1}{x\ln a}$$

I Derivative of Inverse Trigonometry Function

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1} x\right] = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\csc^{-1} x\right] = -\frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\cot^{-1} x\right] = -\frac{1}{1+x^2}$$

J Derivative of Hyperbolic Function

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x \qquad \frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$$

$$\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x \qquad \frac{d}{dx}[\coth x] = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}[\operatorname{csch} x] = -\operatorname{csch} x \coth x$$

K The Second Derivative Test

Suppose f'' is continuous near c.

If $f'(c) = 0$ and $f''(c) > 0$, then f has local minimum at c.

If $f'(c) = 0$ and $f''(c) < 0$, then f has local maximum at c.